Unit 8: Indefinite integration

8.1 Indefinite integration

Teachers may use simple algebraic expressions such as $2x$ and x^2 to bring up the idea that $F(x)$ is a primitive function (or anti-derivative) of $f(x)$ if $F'(x) = f(x)$. Students should be encouraged to determine other primitive functions of $2x$ and should come to realize that finding primitive functions of a given function is a reverse process of differentiation. However the process does not yield a unique function, but a family of functions which differ only by constants. Sketching graphs may be helpful to visualize the notion.

<u>Example 1</u>

Let $f(x) = 3x^2 + 1$, $G(x) = x^3 + x + 1$, $F(x) = x^3 + x + 9$, it is easy to see that $F'(x) = f(x)$ and $G'(x) = f(x)$.

The process of finding the family of primitive functions of a given function is called indefinite integration.

The relation $\int f(x)\,\mathrm{d}x = F(x) + C$ should then be introduced. It should be explained that the integral $\int f(x)\,\mathrm{d}x$ is only a notation, the integral sign $\int$ indicates that the operation of integration is to be performed on the function $f(x)$; dx reinforces the operation is to be performed with respect to the variable x, and $f(x)\,\mathrm{d}x$ does not mean that $f(x)$ is multiplied by dx.

Students should be reminded to find the derivative of the indefinite integrals found and see whether they equal the respective integrands. The meaning of the constant of integration should be explained in detail.

8.2 Some formulae for indefinite integration

Treating indefinite integration as the reverse process of differentiation, students are expected to master the following rules:

(i) $\int k \, \mathrm{d}x = kx + C$, where k is a constant;

(ii) $\int x^n \, \mathrm{d}x = \frac{x^{n+1}}{n+1} + C$, where $n \neq -1$;

(iii) $\int \frac{1}{x} \, \mathrm{d}x = \ln|x| + C$, $x \neq 0$;

(iv) $\int e^x \, \mathrm{d}x = e^x + C$;

(v) $\int kf(x) \, \mathrm{d}x = k\int f(x) \, \mathrm{d}x$, where k is a constant, and

(vi) $\int [f(x) \pm g(x)] \, \mathrm{d}x = \int f(x) \, \mathrm{d}x \pm \int g(x) \, \mathrm{d}x$

<u>Example 1</u>
A T.V. company is launching a promotion campaign aiming to increase the number of viewers, $N(t)$, watching the sports channel 16 days before the opening ceremony of the Olympic Games. Before the campaign, there are 300,000 viewers. The company management expects an increase rate of $\frac{\mathrm{d}N}{\mathrm{d}t} = 100(32 - 2t)$ where $0 \leq t \leq 16$. Determine the number of viewers watching the ceremony when the promotion campaign is enforced.
Students are expected to use the information when t = 0, N = 300,000 to determine the particular function from which 100(32 - 2t) was derived.

8.3 Integration by substitution

Teachers may introduce this section through examples, such as $\int (2x+1)^5 \, \mathrm{d}x$ and $\int 2x\sqrt{x^2+1} \, \mathrm{d}x$.

Different methods of evaluating the same integral may lead to different results, but they only differ by a constant, e.g. $\int (x+1) \, \mathrm{d}x = \frac{(x+1)^2}{2} + C_1$, $\int (x+1) \, \mathrm{d}x = \frac{x^2}{2} + x + C_2$.

It should be noted that integration by parts is not required.