UNIT 12: More about Probability

12.1 The addition rule

Teachers should guide the students to discover the following rule through examples and Venn diagrams:

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Students should be able to apply the above rule to solve problems including cases involving more than two events.

Furthermore if A and B are mutually exclusive, then

$P(A \cup B) = P(A) + P(B)$

12.2 Conditional Probability

Various examples should be used to illustrate the meaning of conditional probability before introducing the definition that

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)},$$

where A can be regarded as the reduced sample space. From the above definition, we can easily derive the rule:

$$P(A \cap B) = P(A)P(B \mid A)$$

<u>Example 1</u>

In a family of two children with at least one girl, what is the probability that the other one is a boy?

<u>Example 2</u>

Suppose a box contains 3 white balls and 5 red balls.

(i) Balls are drawn randomly one by one without replacement from it. What is the probability that the second ball drawn will be red, given that the first ball drawn is white?

(ii) Balls are drawn randomly one by one with replacement from it. What is the probability that the third ball drawn will be white, given that the first two balls drawn are white?

<u>Example 3</u>

A credit card company has surveyed new accounts from university students. Suppose a sample of 160 students indicated the following information in terms of whether the student possessed a credit card X and/or a credit card Y.

	credit card X	
credit card Y	Yes	No
Yes	50	20
No	30	60

Let event A = students possessed two credit cards.
event B = students possessed at least one credit card.
event C = students did not possess any card.
event D = students possessed a credit card X.
event E = students possessed a credit card Y.

Find the probabilities of each of these events A, B, C, D, E, $A \cup B$, $A \cap B$. Find also $P(A|E)$, $P(A|D)$.

Independent event
A and B are said to be independent if one of the following conditions is satisfied:
(i) $P(A|B) = P(A)$
(ii) $P(B|A) = P(B)$
(iii) $P(A \cap B) = P(A)P(B)$

Let A' and B' be the complementary events of A and B respectively. If A and B are independent, then so are A' & B, B' & A, and A' & B'.

<u>Example 4</u>
A fair coin is tossed three times
Let event A = Head appears on first toss.
event B = Head appears on second toss.
event C = Head appears on all three tosses.
To find whether A and B, Band C, C and A are independent.

12.3 Bayes' theorem

Bayes' theorem states that if B_1, B_2, ... are mutually exclusive and exhaustive events then

$$P(B_r \mid A) = \frac{P(A \cap B_r)}{\sum_i P(A \cap B_i)} = \frac{P(A \mid B_r)P(B_r)}{\sum_i P(A \mid B_i)P(B_i)}$$

Students are advised not to memorize the above formula but to calculate the conditional probability from definition with the aid of a **tree diagram.** Teachers should guide the students to do that through examples before deriving the Bayes' theorem.

Example 1

Suppose there are three identical boxes which contain different number of white and black balls.

	Number of white balls	Number of black balls
1 st box	8	3
2nd box	6	5
3rd box	4	7

A box is selected at random and a ball is drawn from it randomly.

(i) What is the probability that a white ball is chosen?

(ii) Suppose a white ball is chosen, find the probability that this white ball comes from the 1st box.

Example 2

The marketing manager of a soft drink manufacturing firm is planning to introduce a new brand of Coke into the market. In the past, 30% of the Coke introduced by the company have been successful, and 70% have not been successful. Before the Coke is actually marketed, market research is conducted and a report, either favorable or unfavorable, is compiled. In the past, 80% of the successful Coke received favorable reports and 40 % of the unsuccessful Coke also received favorable reports. The marketing manager would like to know the probability that the new brand of Coke will be successful if it receives a favorable report.

Example 3

A man decided to visit his friend at North Point. He can reach there by MTR, Bus or Tram respectively. The following information is given:

	Probability of being taken	Probability of being late
MTR	$\frac{5}{8}$	$\frac{1}{4}$
Bus	$\frac{2}{8}$	$\frac{5}{9}$
Tram	$\frac{1}{8}$	$\frac{7}{8}$

(i) He was late for his visit. Find the probability that he had travelled by MTR.

(ii) He was not late for his visit. Find the probability that he had travelled by Bus.