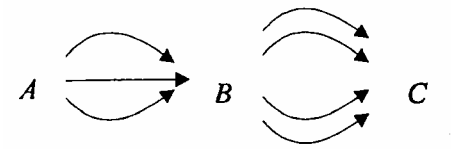
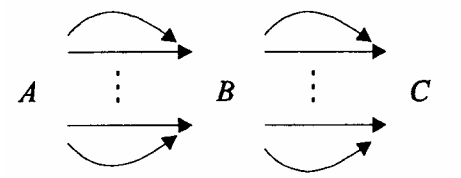
UNIT 1：Permutations and Combinations

1.1 The Fundamental Principle of Multiplication

Teachers may use the following route diagram to illustrate the Fundamental Principle of Multiplication



There are 3 routes from *A* to *B* and 4 routes from *B* to *C*.
How many routes are there from *A* to *C* via *B*?



In general, if there are m routes from *A* to *B* and n routes from *B* to *C*, how many routes are there from *A* to *C* via *B*?

<u>Example 1</u>
Passwords in a computer system:
For a networked computer system, users may set a password to log on a system for security reason. Usually, the password is an *n*-character word or phrase and in fact, each character may be a letter or a number.
How many possible passwords can be set?

Example 2
Using passwords to enter a room or using PIN (Personal Identification Number) through ETM (Electronic Teller Machine) services.

<u>Example 3</u>
Courses of a set dinner menu:
Different combination of different types of soup, main course and dessert.

The Fundamental Principle of Multiplication should be stated and extended to include the case where several operations are performed in succession.

1.2 Definition of permutations

Teachers may take 3 letters, *a*, *b*, *c*, say and consider their arrangements in a row. The symbol *n*! can then be introduced. When the students grasp the concept teacher can further point out that the number of arrangements of *r* different objects chosen from a set of *n* unlike objects is given by P_r^n.

1.3 Definition of combinations

Teachers may take 5 letters, *a*, *b*, *c*, *d*, *e*, say and ask the students to select 3 from them. How many ways of selecting are there? When the students grasp the concept, teacher can further point out that the number of ways of selecting *r* objects from *n* unlike objects is given by C_r^n.

1.4 The symbols *r*!, P_r^n and C_r^n

Students may learn the formulae of P_r^n and C_r^n through the above examples and then the relation $C_r^n = \frac{P_r^n}{r!}$.

The following useful relations should be stated and proved:

$C_r^n = C_{n-r}^n$ (The meaning of this equality should be interpreted by combinatoric approach, i e.

Selecting some is the same as Not selecting the rest)

$C_r^n + C_{r-1}^n = C_r^{n+1}$ (Combinatoric approach ω this relation may be used .However, Direct

verification of the relation by formulae is also acceptable)

1.5 Simple Applications of permutations and combinations to problems

Students are also expected to tackle simple problems involving

(i) The number of ways of arranging *n* unlike objects in a line.

(ii) The number of ways of arranging in a line *n* objects of which *p* are of one type, *q* are of another type, *r* are of a third type, and so on.

However, students are not required to know circular permutations.

The fo1lowing examples only roughly introduce the application of permutations and combinations in some fields. Teachers may raise the problems under different conditions.

Permutations:

(i) The number of arrangements of *n* letters formed from the 26 letters *A* to *Z*.

(ii) The number of ways of hanging n pictures in a line on a wall.

(iii) 4 types of textbooks; Chinese, Mathematics, Physics & Chemistry are arranged in a line on a book shelf. How many ways of arrangement are there?

(iv) 9 team members can be allocated to different positions in a football match. How many ways of arrangement are there?

Combinations:

(i) Mark Six Lottery: the number of ways of selecting 6 numbers from 47 numbers.
(ii) Number of matches (single or double) to be held for *n* participants in a sports game.
(iii) The number of diagonals that can be drawn in a n-sided convex polygon is $\frac{1}{2}n(n-3)$.
(iv) Divide a class of *n* students into *p* groups, each of *q* students.
(v) A team of 4 students is to be selected for a student council. In how many ways can the team be chosen fro111 a short list of 15?